The Derivative

- one of the main concepts in calculus
- is defined as a limit
- occurs frequently in mathematics and science calculations

It is used to determine the slopes of tangent lines and instantaneous velocities.

The derivative of a function $f$ of a number $a$ is

\[
 f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}
\]

This is called taking the derivative using first principles!!!
ex) If \( f(x) = 2x^2 - 5x + 6 \) find \( f(4) \) which is the derivative of \( f \) at 4.

\[
\begin{align*}
\frac{f'(4)}{h} &= \lim_{h \to 0} \frac{f(4+h) - f(4)}{h} \\
&= \lim_{h \to 0} \frac{2(4+h)^2 - 5(4+h) + 6 - (2(4)^2 - 5(4)+6)}{h} \\
&= \lim_{h \to 0} \frac{2(16+8h+h^2) - 20-5h+6 - 32+20}{h} \\
&= \lim_{h \to 0} \frac{32 + 16h + 2h^2 - 20 - 5h + 6 - 32+20}{h} \\
&= \lim_{h \to 0} \frac{11h + 2h^2}{h} = 11 + 2h = 11
\end{align*}
\]

ex) Find the derivative of \( f(x) = x^2 - 3x \) at any number \( a \).

\[
\begin{align*}
\frac{f'(a)}{h} &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h \to 0} \frac{(a+h)^2 - 3(a+h) - (a^2 - 3a)}{h} \\
&= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - 3a - 3h - a^2 + 3a}{h} \\
&= \lim_{h \to 0} \frac{2ah + h^2 - 3h}{h} \\
&= \lim_{h \to 0} \frac{2a + h - 3}{1} = 2a - 3
\end{align*}
\]

Then use the derivative to find the slopes of the tangent lines when \( x = 1, 2, 3, 4 \)

\[
\begin{align*}
x = 1 & \quad m = f'(1) = 2(1) - 3 = -1 \\
x = 2 & \quad m = f'(2) = 2(2) - 3 = 1 \\
x = 3 & \quad m = f'(3) = 2(3) - 3 = 3 \\
x = 4 & \quad m = f'(4) = 2(4) - 3 = 5
\end{align*}
\]
Given a function, $f$, the derivative, $f'$, is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Assignment: p 76 # 4, 5