LESSON 5
Applications of Sinusoidal Data

Outcomes:

* determine a partial equation for a sinusoidal curve given a real world situation
WARM UP

A sine function is given by the equation \( y = 3 \sin 2(x - \frac{\pi}{4}) + 2 \)

a) Without graphing the function, determine the amplitude, phase shift, period, vertical displacement, y-intercept, and domain:

b) Confirm using your calculator.

c) State the x-intercepts from \( 0 \leq x \leq 2\pi \)

* Find these values using technology
WARM UP

Find the equation in the form

\[ y = a \sin b(x - c) + d \]

\[ d = \frac{\text{max} + \text{min}}{2} \]
\[ a = \frac{\text{max} - \text{min}}{2} \]
\[ b = \frac{2\pi}{\text{period}} \]
\[ c = \frac{\text{max} + \text{min}}{2} \]

\[ y = 7.5 \sin (a(x - \frac{\pi}{2})) + 2.5 \]

Period: \( \frac{\pi}{b} = \frac{2\pi}{\pi} = 2 \)

\[ b = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2 \]

Find the equation in the form

\[ y = a \sin b(x - c) + d \]
Application Examples:

1. A Ferris wheel has a radius of 20 m. It rotates once every 40 s. Passengers get on at a point S, which is 1 m above ground level. Suppose you get on at S and the wheel starts to rotate.

   a) Sketch a graph of how your height above the ground varies during the first two cycles.

   ![Graph of height variation]

   b) Write an equation that expresses your height as a function of the elapsed time.

   

   

   \[
   b = \frac{2\pi}{\text{period}} \quad b = \frac{2\pi}{40} = \frac{\pi}{20} \quad c = 10
   \]

   \[
   h = 20\sin\left(\frac{\pi}{20}(t-10)\right)
   \]

   c) Estimate your height above the ground after 45 seconds.

   Use x to find y.

   d) Estimate the first time that you are 35 m above the ground.

   Use y to find x.
2. By using the averages of high and low tide levels, the depth of water, $d(t)$, in metres, in a seaport can be approximated by the sine function,

$$d(t) = 2.4 \sin 0.164\pi(t - 1.5) + 13.4$$

a) Graph the function on your calculator. Select a window that shows maximums and minimums, and 2 cycles of the graph.

b) What is the period of the tide?

$$\text{period} = \frac{2\pi}{0.164\pi} = 12.20$$

c) A cruise ship needs a depth of at least 12 m of water to dock safely. For how many hours per tide cycle can the ship dock safely?

8.81 - .291 = 8.5 hours

Type equation above into y1 and type 12 into y2. Solve for the intersection points in one cycle. Find the difference between the times by subtraction.
3. According to statistical data from Environment Canada, one year in Red Deer the lowest normal daily temperature was -23°C on Jan 26 and the greatest temperature was 27°C on July 26th (day 208).

a) Write a sine and cosine function to approximate the temperatures in Edmonton.

\[
y = 25 \sin \left( \frac{2\pi}{365} (t - 117) \right) + 2
\]

\[
y = 25 \cos \left( \frac{2\pi}{365} (t - 208) \right) + 2
\]

\[
\begin{align*}
\text{a:} & = 25 \\
\text{d:} & = 2 \\
\text{b:} & = \frac{2\pi}{365} \\
\text{c(cos):} & = 208 \\
\text{c(sine):} & = 117
\end{align*}
\]

b) Use your equation to predict the temperature on November 10th, day 324.

-8.2°C

-8.2°C

324

112

3. According to statistical data from Environment Canada, one year in Red Deer the lowest normal daily temperature was -23°C on Jan 26 and the greatest temperature was 27°C on July 26th (day 208).

a) Write a sine and cosine function to approximate the temperatures in Edmonton.

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\end{align*}
\]

b) Use your equation to predict the temperature on November 10th, day 324.

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Solve a Trigonometric Equation Graphically

Ex. Determine the general solutions for the trig equation \[ 16 = 6 \cos \frac{\pi}{6} x + 14 \]

\[ y_1 = 16 \quad y_2 = 6 \cos \left( \frac{\pi}{6} x \right) + 14 \]

1. Get the first 2 solutions!! \[ x = 2.35, x = 9.65 \]
2. Find period. \[ \text{period} = 12 \]
3. Write general solution.
   \[ x = 2.35 + 12n, n \in \mathbb{I} \]
   \[ x = 9.65 + 12n, n \in \mathbb{I} \]
Pg 253 - 254 # 23, 24, 27a,b, C3

Pg 276 - 277 #3, 9(a-d), 10, 19-21